

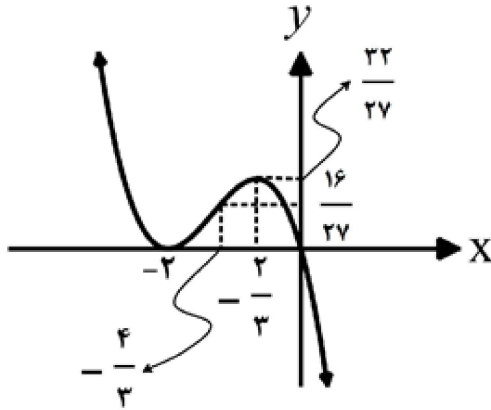
الف) $f(x) = -x(x+2)^2 \Rightarrow x=0 \Rightarrow y=0$

$f'(x) = -1(x+2)^2 + 2(x+2)(-x) = 0$

$(x+2)(-x-2-x) = 0$

$(x+2)(-2x-2) = 0 \Rightarrow \begin{cases} x = -2 \\ x = -\frac{2}{2} \end{cases}$

$f''(x) = 1(-2x-2) + (-2)(x+2) = 0 \Rightarrow f''(x) = -2x-2-2x-4 \Rightarrow -4x-6 = 0 \Rightarrow x = -\frac{3}{2}$



X	$-\infty$	-2	$-\frac{3}{2}$	$-\frac{1}{2}$	$+\infty$
f'	-	+	+	-	
f''	+	+	-	-	
f	↘	↗	↗	↘	

$x = -\frac{b}{3a} = 0 \Rightarrow f(0) = -2$

$a > 0 \Rightarrow$ عطف صعودی

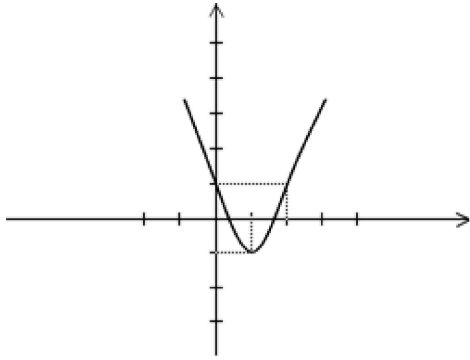
نمودار (ب) صحیح است.

الف) $f(x) = ۳x^۲ - ۴x + ۱ \Rightarrow D = R$

۱) $f'(x) = ۴x - ۴ = ۰ \Rightarrow x = ۱$

۲) $f''(x) = ۴ > ۰$

۳) $x = ۱ \Rightarrow y = ۱$



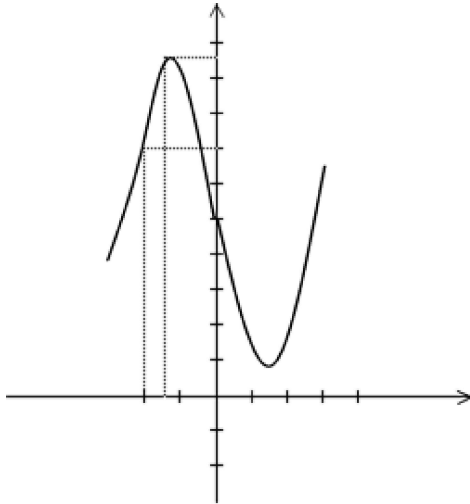
x	$-\infty$	۰	۱	۲
f'	-	-	+	
f''	+	+	+	
f	↘	↘	↗	

کمکی

ب) $f(x) = x^۳ - ۵x + ۵ \Rightarrow D_f = R$

$$f'(x) = ۳x^۲ - ۵ = ۰ \quad \left\{ \begin{array}{l} x = \sqrt{\frac{۵}{۳}} \approx ۱/۳ \\ x = -\sqrt{\frac{۵}{۳}} \approx ۱/۳ \end{array} \right.$$

$f''(x) = ۶x = ۰ \Rightarrow x = ۰$



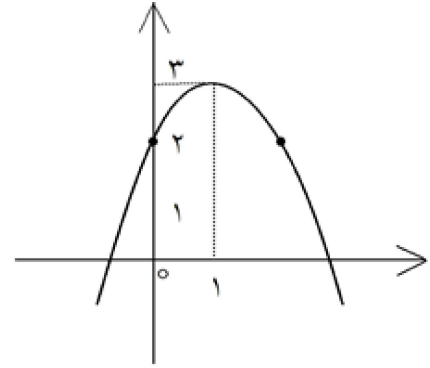
x	$-\infty$	$-\sqrt{\frac{۵}{۳}}$	۰	$\sqrt{\frac{۵}{۳}}$	۲	$+\infty$
f'	+	+	-	-	+	+
f''	-	-	-	+	+	+
f	↗	↗	↘	↘	↗	↗

$\sqrt{\frac{۵}{۳}}$ ۰ $۱/۳$ ۲

$$y' = -2x + 2 = 0 \Rightarrow x = 1 \Rightarrow y = 3 \quad (0/5)$$

4

x	$-\infty$	کمکی 0	1	کمکی 2	$+\infty$
y'		+	0	-	
y	$-\infty$	2 ↗	3	↘ 2	$+\infty$



(0/5)

1

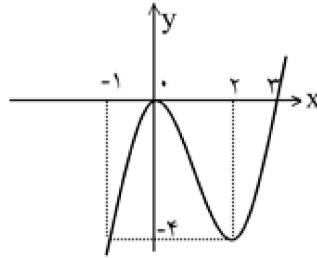
$$y' = 3x^2 - 6x = 0 \begin{cases} x = 0 \Rightarrow y = 0 \\ x = 2 \Rightarrow y = -4 \end{cases}$$

$$y = 0 \Rightarrow x^2 - 2x = 0 \Rightarrow x = 0, x = 2$$

$$x = -1 \Rightarrow y = -4$$

5

x	$-\infty$	-1	0	2	3	$+\infty$
y'		+	0	-	0	+
y	$-\infty$	-4 ↗	0	-4 ↘	0	$+\infty$



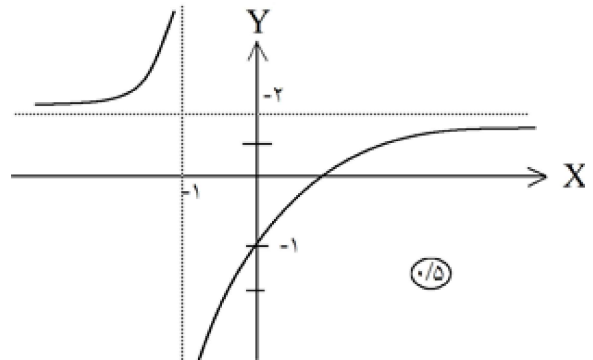
$$\begin{cases} x \rightarrow -1 \Rightarrow x = -1 \text{ مجانب قائم } (0/25) \\ y \rightarrow \infty \end{cases} \text{ و } \begin{cases} x \rightarrow +\infty \Rightarrow y \rightarrow 2 \text{ مجانب افقی } (0/25) \\ y \rightarrow 2 \end{cases}$$

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$$y' = \frac{2}{(x+1)^2} > 0 \quad (0/25), \quad x = 0 \Rightarrow y = -1 \quad (0/25)$$

$$y = 0 \Rightarrow x = \frac{1}{2} \quad (0/25)$$

x	$-\infty$	1	$+\infty$
y'	+	0	+
y	2 ↗	$+\infty$	$-\infty$ ↘ 2

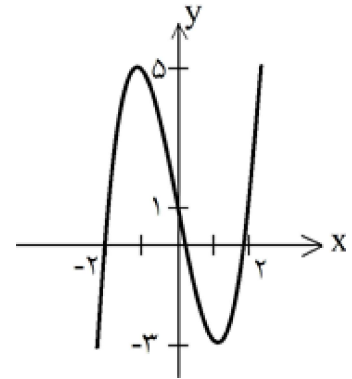


(0/5)

$$y' = 6x^2 - 6 \Rightarrow \begin{cases} x = 1 \rightarrow y = -3 \\ x = -1 \rightarrow y = 5 \end{cases}$$

$$y'' = 12x = 0 \Rightarrow x = 0 \rightarrow y = 1 \text{ نقطة عطف}$$

X	$-\infty$	-٢	-١	٠	١	٢	$+\infty$
y'		+	٠	-	٠	+	
y''	$-\infty$	\nearrow	\nearrow	\searrow	\searrow	\nearrow	$+\infty$
			max		min		



$$x \rightarrow \pm\infty \quad y = \pm\infty$$

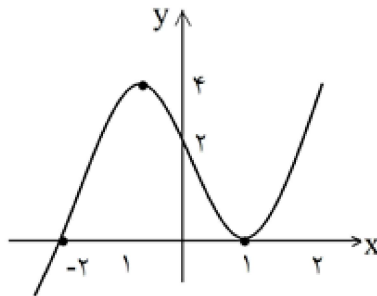
$$y' = (x-1)^2 + 2(x-1)(x+2)$$

$$y' = (x-1)(3x+3) = 0 \Rightarrow \begin{cases} x = 1 \Rightarrow y = 0 \\ x = -1 \Rightarrow y = 4 \end{cases}$$

$$y = 0 \Rightarrow \begin{cases} x = -2 \\ x = 1 \end{cases} \quad y'' = 6x = 0$$

$$x = 0 \Rightarrow y = 2$$

x	$-\infty$	-٢	-١	٠	١	$+\infty$
y'		+	+	-	+	
y	$-\infty$	\nearrow	\nearrow	\searrow	\searrow	$+\infty$
			max		min	



$$y = x^3 - 3x^2 + 2$$

$$y' = 3x^2 - 6x \xrightarrow{y'=0} 3x^2 - 6x = 0 \rightarrow 3x(x-2) = 0 \rightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$$

$$f(0) = (0)^3 - 3(0)^2 + 2 = 2 \text{ و } f(2) = (2)^3 - 3(2)^2 + 2 = -2$$

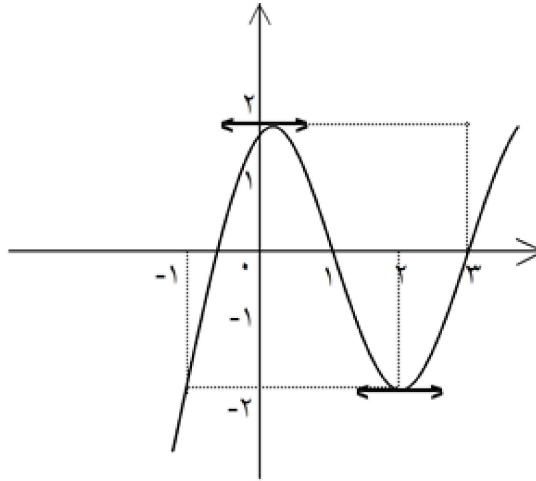
$$y'' = 6x - 6 \xrightarrow{y''=0} 6x - 6 = 0 \rightarrow x = 1$$

$$f(1) = (1)^3 - 3(1)^2 + 2 = 0$$

$$\begin{cases} x \rightarrow \pm\infty \\ y \rightarrow \pm\infty \end{cases}$$

x	$-\infty$	0	1	2	$+\infty$
y'	+		-		+
y	$-\infty$	↗ ↘	↘ ↗	↘ ↗	$+\infty$

Max
نقطه عطف
Min



x	y
$y' = 0$	0
	2
	-2
$y'' = 0$	$\pm\infty$
	1
	0

$$y' = 3x^2 - 6x = 0 \Rightarrow \begin{cases} x = 1 \Rightarrow y = -2 \\ x = -1 \Rightarrow y = 2 \end{cases}$$

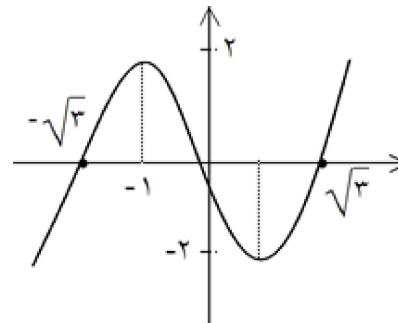
$$x = 0 \Rightarrow y = 2$$

$$y = 0 \Rightarrow \begin{cases} x = 0 \\ x^3 = 3 \Rightarrow x = \pm\sqrt[3]{3} \end{cases}$$

$$y' = 6x = 0 \Rightarrow x = 0, y = 0 \text{ نقطه عطف}$$

نقاط بحرانی $(-1, 2), (1, -2)$

x	$-\infty$	$-\sqrt[3]{3}$	-1	0	1	$\sqrt[3]{3}$	$+\infty$
y'	+	+	0	-	0	+	+
y	$-\infty$	↗ ↘	↘ ↗	↘ ↗	↘ ↗	↘ ↗	$+\infty$



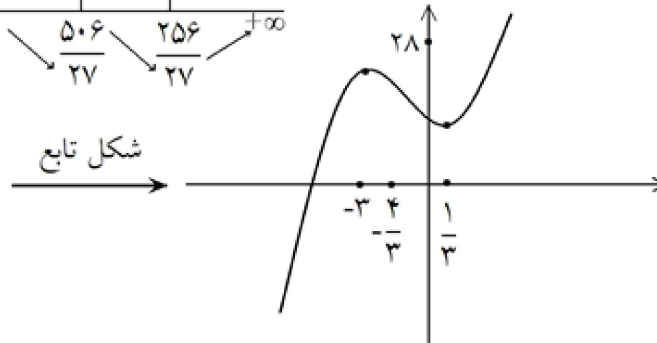
$$y = x^3 + 4x^2 - 3x + 10 \Rightarrow y' = 3x^2 + 8x - 3 = 0 \quad \begin{cases} \text{اکسترمم} \\ x = -3 \\ x = \frac{+1}{3} \end{cases}$$

$$y'' = 6x + 8 = 0 \Rightarrow x = \frac{-4}{3}$$

جدول تغییرات :

x	-∞	-3	-4/3	1/3	+∞	
y'		+	0	-	0	+
y''		-	-	0	+	+
y		28	50/27	256/27		

(D_f = R) این تابع مجانب ندارد.



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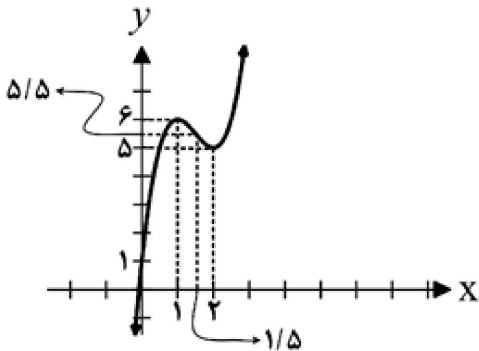
$$(c, 1) = \left(-\frac{d}{c}, \frac{a}{c}\right) \Rightarrow -\frac{d}{c} = c, \frac{a}{c} = 1 \Rightarrow a = c, d = -c^2$$

$$(-1, 0) \Rightarrow a(-1) + b = 0 \Rightarrow a = b \Rightarrow f(x) = \frac{ax+a}{ax-c^2} \Rightarrow f(x) = \frac{x+1}{x-2}$$

$$b) f(x) = 2x^3 - 3x^2 + 12x + 1 \Rightarrow D_f = R$$

$$f'(x) = 6x^2 - 6x + 12 = 0 \Rightarrow \begin{cases} x = 1 \\ x = 2 \end{cases}$$

$$f''(x) = 12x - 6 = 0 \Rightarrow x = \frac{1}{2}$$



	کمکی				
x	-∞	1	1/2	2	+∞
f'	+	+	-	-	+
f''	-	-	-	+	+
f	↗	↗	↘	↘	↗
		16	11/2	5	

$$f'(2) = 0 \Rightarrow f'(x) = 2x^2 + 2bx \Rightarrow b = -2$$

$$f(2) = 1 \Rightarrow 8 + (-12) + d = 1 \Rightarrow d = 5$$

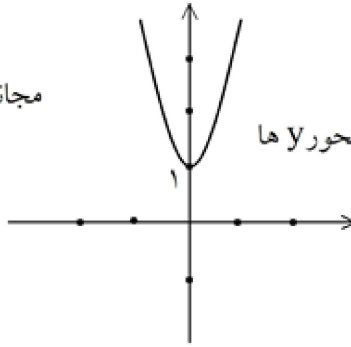
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$$y = x^3 + x^2 + 1 \Rightarrow y' = 3x^2 + 2x = 2x(2x + 1) = 0 \Rightarrow x = 0 \text{ اکسترمم}$$

$$y'' = 6x + 2 > 0 \text{ عطف ندارد.}$$

x	0		
y'	-	0	+
y''	+		+
y	+∞ ↘ ↗ +∞		

مجانب ندارد.



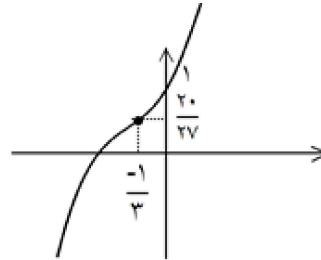
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$$y' = 3x^2 + 2x + 1 > 0$$

$$x = -\frac{1}{3} \text{ نقطه‌ی عطف}$$

$$x = 0 \rightarrow y = 1$$

x	-∞	$-\frac{1}{3}$	0	+∞
y'	+	+	+	
y	-∞ ↗	$\frac{20}{27}$	1 ↘	+∞ ↗

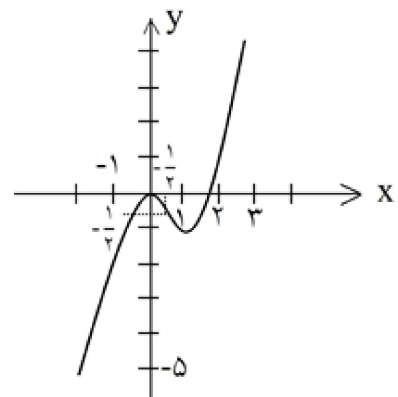


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$$y' = 2x(2x - 2) + 2x^2 = 4x^2 - 2x = 0 \Rightarrow \begin{cases} x = 0 \text{ و } y = 0 \\ x = 1 \text{ و } y = -1 \end{cases}$$

$$y'' = 4(2x - 1) = 0 \Rightarrow x = \frac{1}{2} \Rightarrow \left(\frac{1}{2}, -\frac{1}{2}\right) \text{ نقطه عطف}$$

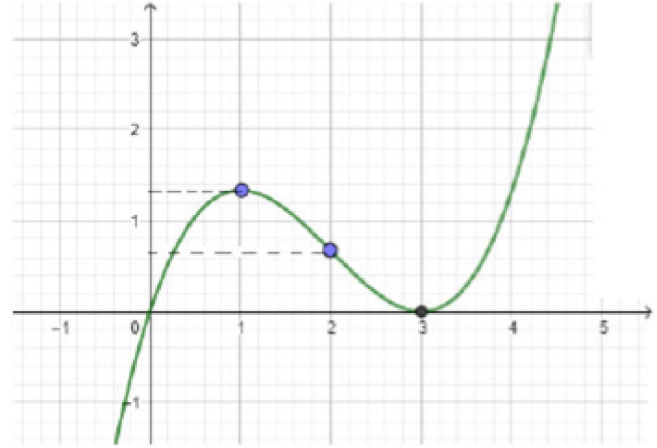
x	-∞	-1	0	$\frac{1}{2}$	1	2	+∞
y'	+		0	-	0	+	
y''	-∞ ↗	-5 ↘	max	$-\frac{1}{2}$	min	4 ↗	+∞ ↗



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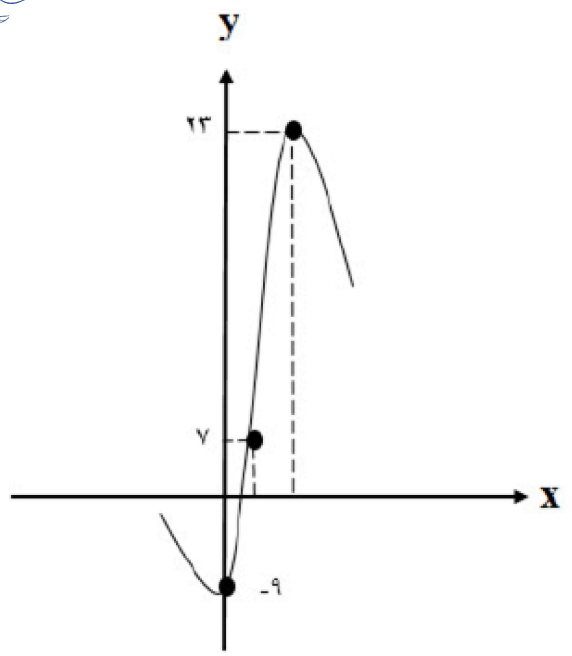
$$f'(x) = x^2 - 4x + 3$$

$$f''(x) = 2x - 4$$



x	$-\infty$	۱	۲	۳	$+\infty$	
$f'(x)$	+	○	-	-	○	+
$f''(x)$			○			
$f(x)$	$-\infty$	$\nearrow \frac{4}{3}$	$\searrow \frac{2}{3}$	$\searrow 0$	$\nearrow +\infty$	
		Max نسبی	نقطه عطف	Min نسبی		

(ص ۱۳۹)



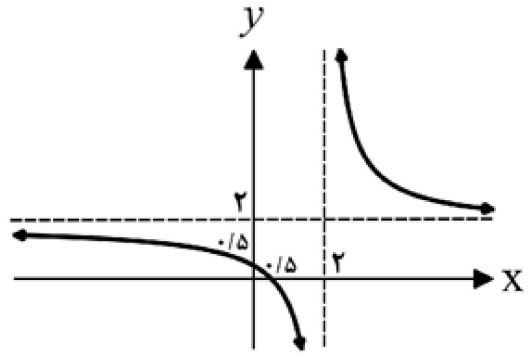
$$f(x) = -x^3 + 7x^2 - 9x, D_f = R$$

$$f'(x) = -3x^2 + 14x = 0 \Rightarrow \begin{cases} x = 0 \\ x = \frac{14}{3} \end{cases}$$

$$f''(x) = -6x + 14 = 0 \Rightarrow x = \frac{14}{6} = \frac{7}{3}$$

X	$-\infty$	0	$\frac{7}{3}$	$\frac{14}{3}$	$+\infty$	
f'	-	+	+	-	-	
f''	+	+	-	-	-	
f	$+\infty$	\searrow	\nearrow	\nearrow	\searrow	$-\infty$
	U	-9	U	V	18	U
		min			max	

ب) $f(x) = \frac{x-1}{x-2} \Rightarrow D = R - \{2\}$
 ۱) $x = 2 \Rightarrow \lim_{x \rightarrow 2} f(x) = \pm \infty$ مجانب قائم
 $\lim_{x \rightarrow \pm \infty} f(x) = \lim_{x \rightarrow \pm \infty} \frac{x-1}{x-2} = 1 \Rightarrow y = 1$ مجانب افقی
 ۲) $f'(x) = \frac{1(x-2) - (x-1)}{(x-2)^2} = \frac{-3}{(x-2)^2} < 0$
 ۳) $f''(x) = \frac{0 + 6(x-2)}{(x-2)^3} = \frac{6}{(x-2)^2}$
 $x - 2 = 0 \Rightarrow x = 2$



x	$-\infty$	۰	۲	$+\infty$
f'	-	-	-	-
f''	-	-	+	+
f	$1 \searrow$	$-\infty \searrow$	$+\infty \searrow$	1

کمکی
 از نقاط کمکی دیگری هم می‌توان استفاده کرد.

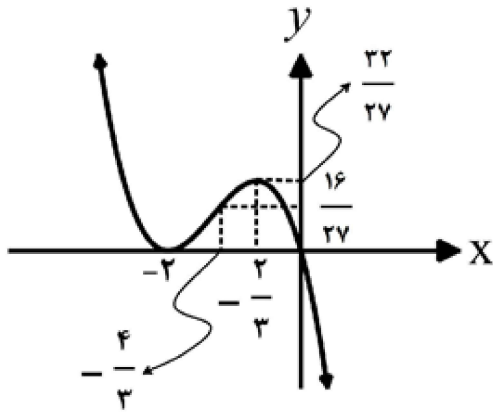
الف) $f(x) = -x(x+2)^2 \Rightarrow x = 0 \Rightarrow y = 0$

$f'(x) = -(x+2)^2 + 2(x+2)(-x) = 0$

$(x+2)(-x-2-2x) = 0$

$(x+2)(-3x-2) = 0 \Rightarrow \begin{cases} x = -2 \\ x = -\frac{2}{3} \end{cases}$

$f''(x) = 1(-3x-2) + (-3)(x+2) = 0 \Rightarrow f''(x) = -3x-2-3x-6 \Rightarrow -6x-8 = 0 \Rightarrow x = -\frac{4}{3}$



x	$-\infty$	-2	$-\frac{4}{3}$	$-\frac{2}{3}$	$+\infty$
f'	-	+	+	-	
f''	+	+	-	-	
f	↘	↗	↗	↘	

$x = -\frac{b}{3a} = 0 \Rightarrow f(0) = -2$

$a > 0 \Rightarrow$ عطف صعودی

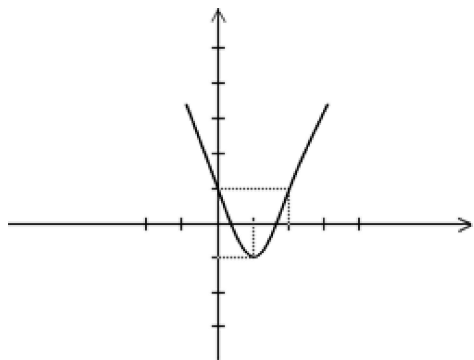
نمودار (ب) صحیح است.

الف) $f(x) = x^2 - 4x + 1 \Rightarrow D = R$

۱) $f'(x) = 2x - 4 = 0 \Rightarrow x = 2$

۲) $f''(x) = 2 > 0$

۳) $x = 2 \Rightarrow y = 1$



x	$-\infty$	۰	۱	۲
f'	-	-	+	+
f''	+	+	+	+
f	↘	↘	↗	↗

ب) $f(x) = x^3 - 5x + 5 \Rightarrow D_f = R$

$f'(x) = 3x^2 - 5 = 0 \Rightarrow \begin{cases} x = \sqrt{\frac{5}{3}} \approx 1/3 \\ x = -\sqrt{\frac{5}{3}} \approx 1/3 \end{cases}$

$f''(x) = 6x = 0 \Rightarrow x = 0$

